what's the point of...

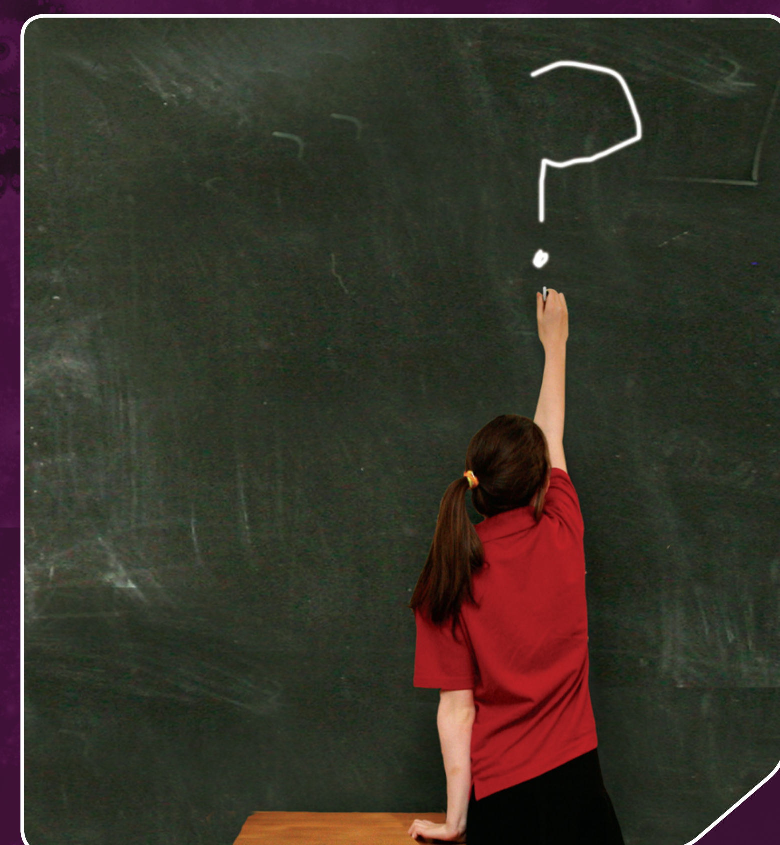
IMAGINARY NUMBERS?

Solving equations

How can you have a number that is imaginary? It sounds like mathematicians are indulging in some wishful thinking!

Actually, when you think about it, negative numbers (and even zero!) are just made-up numbers but they are extremely helpful for describing and solving maths problems. An imaginary number involves the square root of negative one. For hundreds of years mathematicians insisted that you cannot have the square root of a negative number but, in 1545, the Italian mathematician Gerolamo Cardano decided to pretend that there was such a number. (We now call the square root of negative one i.) To his surprise, he found that this new pretend number obeyed the same rules of arithmetic as real numbers and was useful when solving maths problems.

Solving an equation like $x + 5 = 2$ requires you to use the made-up number –7. Solving an equation like $x^2 = -9$ requires you to use the imaginary number 3i. Complicated equations such as $x^3 + 3x^2 - 12x - 18 = 0$ do have real solutions such as $x = 3$ but, to get these answers, during the working-out you use imaginary numbers (in this case, the numbers $2 + 11i$ and $2 - 11i$ appear).



Imaginary numbers are like an off-road detour when the normal road is blocked. When you reach a calculation that you can't do with normal real numbers, imaginary numbers can take you off-road around the problem before bringing you back on to the real road on the other side of the blockage.

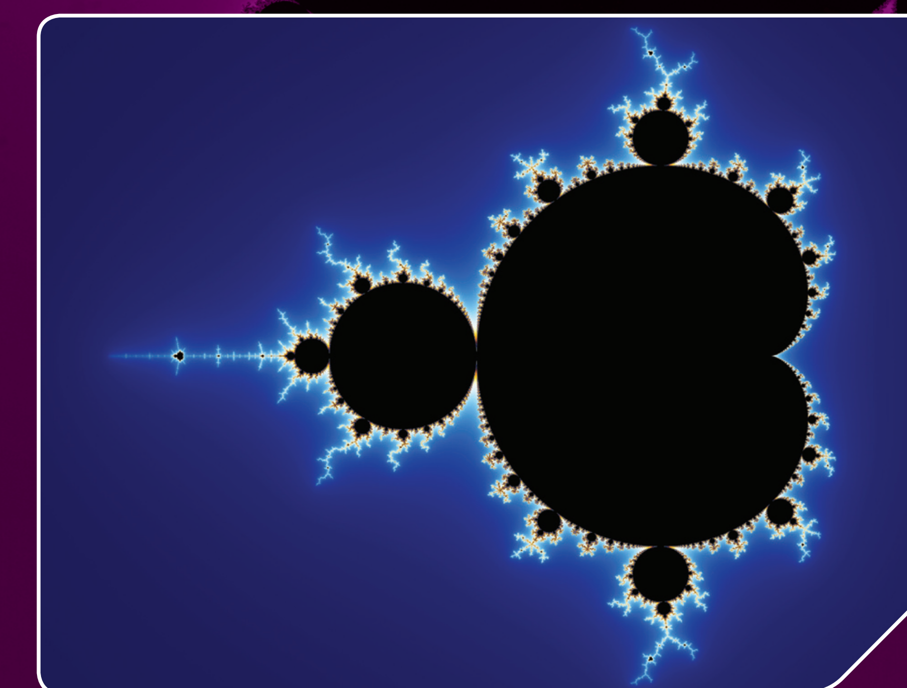
Getting complex

If a number is a combination of a real number and an imaginary number, such as 2 + 11i, we call it a complex number.

Making pictures

Because a complex number has two parts, one real and one imaginary, it can be represented on a diagram as a point with the real part giving its horizontal position and the imaginary part its vertical position (a bit like coordinates).

Such a diagram is known as an Argand diagram after the French mathematician Jean-Robert Argand. In 1978 the mathematicians Robert Brooks and Peter Matelski decided to try repeatedly squaring complex numbers and then adding the original number. They then drew an Argand diagram using these complex numbers and coloured in all the ones that stayed small no matter how long they continued the squaring and adding process. What they ended up with is the Mandelbot set, a picture of which is shown on the right.



Electronics

It was noticed by the mathematician Leonhard Euler in 1748 that imaginary numbers were very good at describing things that rotate or oscillate.

Many years later, electrical engineers were trying to find a good way to mathematically represent the alternating currents that power our modern electronic lifestyle. They realised that they could represent the current flowing as the real component of a more complicated imaginary function. While it may sound like this would make things more complicated, doing calculations with the imaginary function was far easier than just using ordinary numbers – particularly when looking at electrical phase change and impedance. At the end of the imaginary calculation, the answer they needed was just the real component.

In electronics, the symbol i was already used, to represent current, so engineers use j to represent the square root of negative one.

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